

# Cosmological constant and spontaneous gauge symmetry breaking: the particle physics and cosmology interface charade

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## Abstract

We describe one of the remarkable problems of theoretical physics persevering up to the beginning of the millennium. All gauge theories with spontaneous gauge symmetry breaking from the standard model of particle physics with the electroweak symmetry breaking at the Fermi scale, 246 GeV, up to strings, supergravity, and the M(embrane)-theory superunification with symmetry breaking starting near the Planck scale,  $10^{19}$  GeV, foresee that the spontaneous symmetry breakings induce a vacuum energy at least 50 orders of magnitude larger than the stringent experimental bound  $G\Lambda \lesssim 10^{-122}$  on the value of the cosmological constant  $\Lambda$ . This fact seems to have a universal character since it occurs from the Fermi scale up to the Planck one. It is the vacuum catastrophe.

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## I. INTRODUCTION

According to the general relativity [1,2], the vacuum energy density has a defined meaning since it couples unavoidably with gravitation and can be parametrized by a magnitude known as cosmological constant,  $\Lambda$ . The cosmological bounds on  $\Lambda$  are severe and as we will see they result in the smallest number of physics directly related to the fundamental universal interactions. In the year of 1922, A.A. Friedmann [3] (1888–1925) demonstrated that the Einstein's general relativity equations admit non-static solutions connected to an expanding Universe. In a first stage, Einstein even demonstrated that the argument conducing to the conclusion of Friedmann is wrong [4]. Nevertheless, he found out his own mistake [5] and started to consider the Friedmann's theoretical results as clarifying. With the discovery of the expansion of the Universe by Edwin P. Hubble [6] (1889–1953), according to whom there is a linear relation between velocities and distances at cosmological scale, Einstein disregarded definitely the cosmological constant and confirmed besides W. de Sitter [7] (1872–1934) that this term is completely unsatisfactory theoretically.

The standard model of the non-gravitational interactions [8] with the internal symmetry gauge semisimple group

$$\mathcal{G}_{321} \equiv \text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y \quad (1)$$

has resisted to the experimental challenges [9] being proved in tests of accuracy with great success [10]. It was also verified the experimental indication of incompleteness of the standard model related to the fermionic nature of the three neutrino flavors. It has been noticed that there is oscillation among the different neutrino flavors [11] which can happen whether the fermion is described by a four components Dirac [12] spinor or by the two components Majorana [13] state and not only by a pair of Weyl [14] eigenstates of chirality,

$$\gamma_5\psi = -\psi, \quad \bar{\psi}\gamma_5 = \bar{\psi} \quad (2)$$

where  $\gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ ,  $i = \sqrt{-1}$ , and  $\gamma^\mu$ ,  $\mu = 0, 1, 2, 3$  are the Dirac matrices. The chiral invariance does not reveal itself since any fundamental fermion is massive and a Dirac mass term  $m_\psi\bar{\psi}\psi$  violates the chiral invariance due to an algebraic sign change. The good symmetry to provide the dynamics is the *local* gauge symmetry related to the properties of the gauge bosons which transmits the interactions instead of the *global* symmetries such as the U(1) symmetry associated with the conservation of lepton number or with the baryon charge. Nevertheless, the electromagnetic U(1) local symmetry, associated with the conservation of the electric charge, is a local gauge symmetry and not a global one, whose group dimension is one and thus there is a gauge boson associated, the photon, and as the group rank is also one this gauge boson is electrically neutral and is also its own antiparticle.

The origin of the unified description of the dynamics of the fundamental interactions retrace to the pioneer attempts; in 1914 of G. Nordström [15] (1881–1923) and, soon later, still in the twenties, of T. Kaluza [16] (1885–1954) and O. Klein [17] (1849–1925). Originally it was proposed the unification of the Maxwell theory of the electrodynamics with the general relativity in a space-time of five dimensions. The fifth dimension would be compactified in the scale of gravity quantization which is the Planck scale,  $\sim 10^{19}$  GeV  $\simeq 10^{-35}$  m, where 1 GeV (gigaelectron volt)  $\equiv 10^9$  eV with 1 eV  $= 1.602 \times 10^{-19}$  J, being therefore undetectable nowadays. The fifth dimension  $x_5$  has a circle topology,

$$\phi(x_5, r) = \phi(x_5 + 2\pi r), \quad (3)$$

where  $r \simeq 10^{-35}$  m. The non-trivial generalization of the space-time Kaluza–Klein extension to the space of internal symmetries result in the Yang–Mills [18] theories built up in a space with  $((3+1)+N)$  dimensions decomposed as the product of the Minkowski flat space-time,  $M_{(3+1)}$ , with a manifold  $G$  of dimension  $N$  and so  $M_{(3+1)} \otimes G$  is the complete gauge group. This is the base of the non-Abelian gauge theories which describe all interactions including the gravitation since the general relativity is a gauge theory par excellence in which the internal symmetry transformations correspond to the general transformations of coordinates. The masses of all the fermions including the neutrinos [11], the gauge bosons, and also of the Higgs scalar boson, the particle undetected so far, are generated in the process of spontaneous breaking of gauge symmetry by the Higgs–Kibble [19,20] mechanism. Always that a spontaneous breaking of the gauge symmetry occurs in the theory, from the standard model of the elementary particles in the Fermi scale,  $\simeq 246$  GeV, up to superstrings [21] in the Planck scale,  $\simeq 10^{19}$  GeV, it is generated a constant term in the scalar potential that corresponds to a great increase in the energy density of the vacuum state. In the realm of the study of the high energy phenomenological processes it is even possible to simply dismiss this term. However, this constant term contributes to the value of the cosmological constant as we could see ahead.

The general relativity [22] is formulated as a classical field theory. All of the attempts of quantization have always resulted in non-renormalizable theories [23]. The purpose is to accomplish the unification of the gravitation with the other interactions so that the infinities appearing in the different sectors cancel themselves order-by-order in the perturbative series, resulting in a renormalizable theory. The same theory would offer the reason why the cosmological constant  $\Lambda$  is so small,

$$G\Lambda \lesssim 10^{-122} \quad (4)$$

where  $G = 6.673(10) \times 10^{-11}$  kg $^{-1}$  m $^3$  sec $^{-2}$  is the Newton–Cavendish universal constant of gravitation. At present, the most successful of the candidates to accomplish this unification is the local supersymmetry [24] containing the gravity, being known as supergravity [25] which is one of the facts of the superstrings [26] theories. Today, the partial, grand, and the complete or total unification possibilities are: compositeness, technicolor, grand unification, symmetric left-right gauge groups, chiral gauge groups, Kaluza–Klein models, supersymmetry, supersymmetric grand unification, supergravity, superstrings [27] and the superunification Membrane-theory [28] with a total unification of all fundamental interactions.

## II. ONE CENTURY AGO: ULTRAVIOLET CATASTROPHE

In June of 1900, Lord Rayleigh [29] (J.W. Strutt, 1842–1919) suggested, for the first time, the application to the thermal bath of a black body of the principle of equipartition of energy by the Maxwell–Boltzmann distribution,

$$f(v, T) = \frac{dN}{dv} \propto v^2 \exp\left\{-\frac{mv^2}{2kT}\right\} \quad (5)$$

which is the statistical distribution of an ensemble with  $N$  indistinguishable particles of any kind with mass  $m$  of a system in equilibrium at a temperature  $T$ , where  $k = 1.381 \times 10^{-23} \text{ J K}^{-1}$  is the Boltzmann constant. The function  $f(v, T)$  will be zero when  $v = 0$ , will reach the maximum value when  $v = (2kT/m)^{\frac{1}{2}}$  and will reduce rapidly until zero with the ulterior increase of the velocity. The Maxwell–Boltzmann distribution has a universal character but it is founded on the classical physics so that its application could not be appropriate to quantum systems of fermions or bosons. Actually, the classical distribution of Maxwell–Boltzmann when applied to a cavity of harmonic oscillators does not provide the correct energy density but

$$\rho(\nu, T) = \frac{8\pi}{c^3} \nu^2 kT \quad (6)$$

which is a function of the square of the frequency. This turned to be recognized as the Rayleigh–Jeans formula. The  $\nu^2 T$  law was also obtained by Lorentz [30] and Einstein [31] in 1905. The divergent behavior of  $\rho(\nu, T)$  for high frequencies was named by Paul Ehrenfest (1880–1933) as ‘ultraviolet catastrophe’ in 1911 [32]. Lord Rayleigh, in his work of 1900 [29] does not calculate the constant

$$c_1 = \frac{8\pi}{c^3} k \quad (7)$$

in

$$\rho(\nu, T) = c_1 \nu^2 T \quad (8)$$

but in order to supply the catastrophic divergent behavior at high frequencies he introduced *ad hoc* a cutoff exponential factor thereby suggesting the radiation law

$$\rho(\nu, T) = c_1 \nu^2 T \exp\{c_2 \nu/T\} \quad (9)$$

known as Rayleigh law. In the year of 1905 he retake his  $\nu^2 T$  law, then determining the  $c_1$  constant but obtaining  $c_1/8$ . The mistake is corrected by Sir James Hopwood Jeans [33] (1887–1946) who is thanked by Lord Rayleigh [34] by his contribution.

It is not known whether M. Planck [35] (1858–1947) knew of the June work of Lord Rayleigh. Anyhow he does not mention such article, neither does Lorentz [30]. However it is worthy to notice that Planck made reference to the inspiration he had received from the statistical methods of L. Boltzmann [36] (1844–1916). The job accomplished by Planck was to begin with the Maxwell–Boltzmann classical distribution but treating the energy content of the stationary electromagnetic waves in the cavity of a black body as a discrete magnitude replacing the integral continuous summation by a discrete sum indexed with the principal quantum number in the energy density distribution. The Planck job is well known and is called *quantization*. Immediately, he got the spectral density

$$\rho(\nu, T) = \frac{8\pi}{c^3} \nu^2 \frac{h\nu}{\exp\{h\nu/kT\} - 1} \quad (10)$$

where  $h = 6.626\,068\,76(52) \times 10^{-34} \text{ J sec}$  is the Planck constant [37]. Under the condition that  $h\nu/kT \gg 1$  the Planck distribution fall back in the W. Wien [38] (1864–1928) law from 1896,

$$\rho(\nu, T) = \frac{8\pi}{c^3} \nu^2 [h\nu \exp\{-h\nu/kT\}] \quad (11)$$

which is also correct in the quantum realm,  $h\nu/kT \gg 1$ , as constated for the first time by the remarkable experiment of F. Pashen [39] (1865–1947) in which  $h\nu/kT \simeq 15$  for  $T = 10^3$  K and  $\lambda = 1\mu\text{m} = 10^{-6}$  m.

In 1906, Einstein [40] realized that the Planck theory of 1900 uses implicitly the hypothesis of the quantum of radiation that is a quantum property of free electromagnetic radiation. The energy of an oscillator of the black body electromagnetic thermal bath could take only values which are integer multiples of  $h\nu$ . In the processes of emission and absorption the energy of an oscillator varies only by integer multiples  $n h\nu$  of the quantum  $h\nu$ , being  $n$  the principal quantum number related to the quantization of energy. Hence, the photon is a state of the electromagnetic field with a frequency  $\nu$  and a wave vector  $\mathbf{k}$  well defined linked, respectively, to the energy

$$E = h\nu \quad (12)$$

and to the linear momentum

$$\mathbf{p} = h\mathbf{k} \quad (13)$$

which satisfies the relativistic equation of energy

$$E = c|\mathbf{p}| \quad (14)$$

for a massless particle. The word *photon* appeared for the first time in an article of 1926 by Gilbert Lewis [41].

It is interesting to notice that both Planck and Einstein and also Lorentz presented serious restrictions in relation to the quantum of action which retains its individuality in the propagation [36]. Einstein, in a letter [36] of 1951, wrote ‘All these fifty years of meditation have not approached me to the answer to the question: *What are the quanta of light?*’

### III. THE STANDARD MODEL OF NON-GRAVITATIONAL FUNDAMENTAL INTERACTIONS

The standard model of the non-gravitational fundamental interactions, which are the two nuclear ones, the ‘weak’ and the ‘strong’ interactions, and the electromagnetic interaction consists of three relativistic quantum gauge field theories with the corresponding gauge symmetries  $SU(3)_c$  and  $SU(2)_L \times U(1)_Y$  associated to the quantum chromodynamics [8] (QCD) and, in the electroweak sector  $SU(2)_L \otimes U(1)_Y$ , to the quantum flavor dynamics [8] (QFD) and to the quantum electrodynamics [42] (QED), the only one among the three ones which is an Abelian theory. The gauge coupling constants of the electroweak sector,  $g$  of  $SU(2)_L$  and  $g'$  of  $U(1)_Y$ , and the parameter  $\theta_W$ ,  $\tan \theta_W = g'/g$ , are all free parameters, so that there is no (grand) unification of the interactions even in the called electroweak sector. The c-number  $Y$  that indexes the Abelian factor  $U(1)_Y$  is the weak hypercharge related to the electric charge operator

$$\frac{\mathcal{Q}}{|e|} = T_3 + T_0 \quad (15)$$

where  $T_3 = \frac{1}{2}\text{diag}(+1, -1)$  is one half of the third Pauli traceless matrix and  $T_0 = Y\frac{1}{2}\text{diag}(+1, +1)$ .

The fundamental chiral fermions are grouped in three generations of leptons and quarks which are the fundamental constituents of all matter. To any fermion  $\psi$  we define its chiral left-handed ( $L$ ) component

$$\psi_L = P_L \psi \quad (16a)$$

and the right-handed ( $R$ ) one

$$\psi_R = P_R \psi \quad (16b)$$

where

$$P_L = \frac{1}{2}(1 - \gamma_5), \quad (17a)$$

$$P_R = \frac{1}{2}(1 + \gamma_5) \quad (17b)$$

are the chiral idempotent projectors. Counting three  $SU(3)_c$  color charges for each quark flavor each generation contains 15 Weyl fermions in two chiral states,  $L$  and  $R$ , which are attributed to the *fundamental representation* of the  $\mathcal{G}_{321}$  gauge group. For three families of fermions there are 45 massless Weyl fermions which under the gauge group are attributed to the following representations:

Chiral left-handed leptons transform under  $\mathcal{G}_{321}$  as

$$\Psi_{\ell L} = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L \sim (\mathbf{1}_c, \mathbf{2}_L, Y = -1) \quad (18a)$$

while chiral right-handed components transform as singlets under all the factors of the gauge semisimple group,

$$\ell_R \sim (\mathbf{1}_c, \mathbf{1}_R, Y = -2) \quad (18b)$$

for the  $\ell = e^-, \mu^-, \tau^-$  flavors. Three families of left-handed chiral quarks are attributed to color triplets but also to  $SU(2)_L$  flavor doublets

$$\Psi_{qL} = \begin{pmatrix} \mathcal{U}_q \\ \mathcal{D}_q \end{pmatrix}_L \sim (\mathbf{3}_c, \mathbf{2}_L, Y = 1/3) \quad (19a)$$

with the respective flavor  $R$ -singlets

$$\mathcal{U}_{qR} = \{u_{qR}, c_{qR}, t_{qR}\} \sim (\mathbf{3}_c, \mathbf{1}_R, Y = 4/3) \quad (19b)$$

$$\mathcal{D}_{qR} = \{d_{qR}, s_{qR}, b_{qR}\} \sim (\mathbf{3}_c, \mathbf{1}_R, Y = -2/3) \quad (19c)$$

where  $q = 1, 2, 3$  denotes the  $SU(3)_c$  color index (*red, green, blue*) and  $\mathcal{U}_{qR}$  and  $\mathcal{D}_{qR}$  denote the three flavors with electric charges  $\pm \frac{2}{3}|e|$  and more three corresponding flavors with the electric charges  $\mp \frac{1}{3}|e|$  for the particle and antiparticle states. It is contained in the inner of a same family of quarks 12 Weyl two-component spinors corresponding to two flavors, type ‘up,’  $\mathcal{U}_q$ , and ‘down,’  $\mathcal{D}_q$ , with two states of chirality,  $L$  and  $R$ , and still three flavor degrees of freedom. Therefore, counting the three spinors of the leptonic sector, it results in an amount of 15 Weyl spinors.

Otherwise, the entire sector of gauge bosons is attributed to the *adjoint representation* of the  $G_{321}$  group. The color factor,  $SU(3)_c$ , with a dimension  $3^2 - 1 = 8$ , contains eight generators which are the  $3 \times 3$  Gell-Mann matrices closing among them the Lie algebra

$$[\lambda_a, \lambda_b] = 2i f_{abc} \lambda_c, \quad a = 1, 2, \dots, 8 \quad (20)$$

associated to eight gauge bosons, the gluons  $g_\mu^a$ , which are the transporter of the strong nuclear interaction among hadrons, and also to eight non-vanishing group structure constants  $f_{abc}$ . The rank of the color group is  $3 - 1 = 2$ , corresponding to the number of generators that are diagonal in the matricial representation. Likewise, in the case of the  $SU(2)_L$  flavor group the dimension  $2^2 - 1 = 3$  correspond to the three group generators which are one half times the  $\sigma_k$  Pauli matrices satisfying the Lie algebra

$$[\sigma_k, \sigma_l] = 2i \epsilon_{klm} \sigma_m, \quad k = 1, 2, 3 \quad (21)$$

and to the symmetry eigenstates gauge bosons,  $W_\mu^k$ , and, finally, to the unique generator,  $Y/2$ , of the  $U(1)_Y$  Abelian factor with the gauge boson  $B_\mu$ . The mass eigenstates physical states are a pair of electrically charged gauge bosons

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2) \quad (22a)$$

with mass [37]  $80.419(56)$  GeV/ $c^2$  and the neutral gauge bosons, the photon,

$$A_\mu = \cos \theta_W W_\mu^3 + \sin \theta_W B_\mu \quad (22b)$$

and

$$Z_\mu = -\sin \theta_W W_\mu^3 + \cos \theta_W B_\mu \quad (22c)$$

with mass [37]  $91.1882(22)$  GeV/ $c^2$  in which, as masses, the parameter  $\sin^2 \theta_W = 0.23147(16)$  is one of the twenty free parameters of the standard model. Almost a half of the free parameters of the whole standard model are masses.

It can be realized now that each fermion family contains four chiral quarks. Nonetheless, the leptons of each family consist only of three Weyl spinors,  $\ell_L$ ,  $\ell_R$ , and  $\nu_{\ell L}$ . What is the reason of this asymmetry between leptons and quarks? This difference is the reason of the neutrino physics and of the fact that the electroweak sector  $SU(2) \otimes U(1)$  to have the chiral character  $SU(2) = SU(2)_L$  and not the symmetric one [43]  $SU(2) = SU(2)_L \otimes SU(2)_R$ . Oscillations of neutrino flavors were confirmed [11], indicating that they could be massive fermions so that the independent Weyl pairs  $(\nu_{\ell L}, \bar{\nu}_{\ell R}^c)$  and  $(\nu_{\ell R}, \bar{\nu}_{\ell L}^c)$  involving the charge conjugation operation,

$$(\nu_{\ell L})^c = (P_L \nu_{\ell})^c = \frac{1}{2}(1 + \gamma_5)\gamma^0 \nu_{\ell}^* = (\nu_{\ell}^c)_R, \quad (23a)$$

and

$$(\nu_{\ell R})^c = (P_R \nu_{\ell})^c = \frac{1}{2}(1 - \gamma_5)\gamma^0 \nu_{\ell}^* = (\nu_{\ell}^c)_L \quad (23b)$$

turn to be connected. All the fundamental fermions of the standard model have mass and are Dirac fermions, say  $\psi$ , in four states,  $\psi_L$ ,  $\psi_R$  and the charge conjugated states  $(\psi_L)^c = (\psi^c)_R$  and  $(\psi_R)^c = (\psi^c)_L$  corresponding to the antiparticles. To each Dirac fermion we define the charge conjugated field

$$\psi^c = C\bar{\psi}^T \quad (24)$$

which also satisfies the Dirac equation,

$$(i\hbar\gamma^\mu\partial_\mu - mc)\psi^c = 0. \quad (25)$$

The neutrinos are the only fundamental fermions that do not have electric charge [44], even though they can have the character of Majorana fermions to which  $\psi^c = (\text{phase factor})\psi$ .

Finally, in relation to the four fundamental interactions there is a curious coincidence. Why ‘4-3-2-1’? Why four dimensions of the space-time? Why SU(3) for the color group? Why SU(2) for the weak isospin group? Why U(1) for the quantum electrodynamics?

#### IV. VACUUM CATASTROPHE

Let us now describe how the standard model triggers the ‘vacuum catastrophe’ [45]. Consider a real scalar field  $\phi$  whose dynamics is contained in the Lagrangian density

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \quad (26)$$

where the first term with derivatives is the kinetic one and the potential is

$$V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 \quad (27)$$

which contains the quadratic mass term and the interaction term  $\lambda\phi^4$ . The Lagrangian  $\mathcal{L}(\phi)$  is invariant under the discrete symmetry transformation  $\mathcal{L}(\phi) \rightarrow \mathcal{L}(-\phi)$ , then  $\mathcal{L}(-\phi) = \mathcal{L}(\phi)$ . In the standard model of elementary particles the more general scalar potential which allows the implementation of the spontaneous gauge symmetry breaking

$$\mathcal{G}_{321} \rightarrow \text{SU}(3)_c \times \text{U}(1)_Q \quad (28)$$

can be written as

$$V(\Phi^\dagger\Phi) = a\Phi^\dagger\Phi + b(\Phi^\dagger\Phi)^2 \quad (29)$$

where  $a$  and  $b$  are two numbers,  $\Phi^\dagger \equiv (\Phi^*)^T = (\Phi^T)^*$ , and

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (\mathbf{1}, \mathbf{2}, Y = +1) \quad (30)$$

is the doublet of scalar fields in the SU(2) fundamental representation. In the process of spontaneous symmetry breaking only the neutral component obtains a vacuum expectation value

$$\langle 0 | \phi^0 | 0 \rangle \equiv \langle \phi^0 \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma \end{pmatrix} \quad (31)$$

so that, in terms of  $\sigma$ , the potential becomes

$$V(\sigma) = \frac{a}{2}\sigma^2 + \frac{b}{4}\sigma^4 \quad (32)$$

and defining

$$a \equiv -m^2, \quad b \equiv \lambda; \quad m > 0 \quad (33)$$

it results

$$V(\sigma) = -\frac{1}{2}m^2\sigma^2 + \frac{1}{4}\lambda\sigma^4. \quad (34)$$

The minimum of this potential is determined by the general conditions

$$V' \equiv \frac{\partial V}{\partial \sigma} = 0, \quad V'' \equiv \frac{\partial^2 V}{\partial \sigma^2} > 0 \quad (35)$$

which in terms of the potential parameters  $m$  and  $\lambda$  are

$$V' = -m^2\sigma + \lambda\sigma^3 = 0, \quad (36a)$$

and

$$V'' = -m^2 + 3\lambda\sigma^2 > 0 \quad (36b)$$

so that the minimum of the potential

$$V(\sigma = \sigma_{\pm}) = -\frac{m^4}{4\lambda} \quad (37)$$

occurs exactly when

$$\sigma_{\pm} = \pm(m^2/\lambda)^{\frac{1}{2}}. \quad (38)$$

On account of the fact that  $V(\sigma_+) = V(\sigma_-)$  either  $V(\sigma_+)$  or  $V(\sigma_-)$  are equivalent minimal values of the potential the symmetry of reflection  $\phi \rightarrow -\phi$  in the Lagrangian  $\mathcal{L}(\phi)$  given in Eq. (26) is broken by the choice of one of the vacuum states. A symmetry of the Lagrangian that is not respected by the vacuum state is a spontaneously broken symmetry. In physics, the word ‘vacuum’ not only has the meaning of ‘empty space’ but also denotes the fundamental state, the state of lower energy in a quantum field theory. In general, the vacuum

state is a Lorentz invariant eigenstate. By choosing a unitary gauge, the doublet of scalar fields can be placed as

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma + H \end{pmatrix} \quad (39)$$

where  $H$  is the Hermitian field associated to the Higgs boson of the electroweak sector of the standard model. Arising out of this, in terms of  $H$ , the scalar potential in Eq. (29) will contain even terms of fourth order,

$$V(H) = -\frac{m^4}{4\lambda} - m^2 H^2 + \lambda \sigma H^3 + \frac{\lambda}{4} H^4 \quad (40)$$

including the quadratic mass term,  $\frac{1}{2}M_H^2 H^2$ , with  $M_H^2 = 2m^2$  but also the constant term  $-m^4/(4\lambda)$  which is the nonvanishing term of the potential when evaluated in the minimum states  $\phi = \sigma_{\pm}$ . It was suggested for the first time by Zel'dovich [46] that this self-energy of vacuum is interpreted as a cosmological constant

$$\Lambda = \frac{8\pi G}{c^4} V(\sigma_{\pm}) \quad (41)$$

in the modified Einstein equations

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} &= \frac{8\pi G}{c^4} T_{\mu\nu} - \Lambda g_{\mu\nu} \\ &= \frac{8\pi G}{c^4} (T_{\mu\nu} - V(\sigma_{\pm})g_{\mu\nu}) \end{aligned} \quad (42)$$

where  $G = 6.673(10) \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$  is the Newton–Cavendish constant being the fundamental universal constant associated to the gravitational interaction. These equations contain the term that involves the cosmological constant  $\Lambda$  whose experimental bound is

$$G\Lambda \lesssim 10^{-122}. \quad (43)$$

Everything in the Universe conspire extremely well to generate and to keep an exceptionally small numerical value. Any phase transition as the spontaneous gauge symmetry breaking is always accompanied by a modification in the vacuum energy which is at least 50 orders of magnitude higher than this limit [47]. The limit that is established to the value of the cosmological constant contained in the Einstein equations of general relativity is the result of this conspiration that constitutes the number  $10^{-122}$ . This number is not zero; however, it is the smallest fundamental number of physics. In order to establish this bound it is necessary the use of *natural units*. It is established the numerical adimensional value  $c = \hbar = 1$  for the velocity of light in the vacuum ( $c$ ) and for the reduced Planck constant ( $\hbar \equiv h/2\pi$ ) which are two universal physical constants associated to the Maxwell electromagnetic theory and the theory of special relativity ( $c$ ), and to the quantum mechanics ( $\hbar$ ). The gravitational constant  $G$  is the universal physical constant involved in the constraint  $G\Lambda \lesssim 10^{-122}$  on the cosmological constant  $\Lambda$ .

Let us now establish the Planck scale through the combination of the  $c$ ,  $G$ , and  $\hbar$  universal constants. The Planck energy scale is  $E_P = (\hbar c^5/G)^{\frac{1}{2}} \simeq 1.2211 \times 10^{19} \text{ GeV}$ . However, Planck

had the idea that the universal physical fundamental constants are enough to determine natural units of energy (GeV), but also scales of lenght,  $l_P = (\hbar G/c^3)^{\frac{1}{2}} \simeq 1.6161 \times 10^{-35}$  m and of time,  $t_P = (\hbar G/c^5)^{\frac{1}{3}} \simeq 5.3904 \times 10^{-44}$  sec.

The high energy physics studies the quantum and relativistic regimes of nature. We always try to seek events among elementary particles which involves velocities near to that of light in vacuum  $c$ , and actions or angular momenta of the order of the reduced Planck constant,  $\hbar$ . This is the reason why it is tacitly placed  $c = \hbar = 1$  which makes the high energy physics unidimensional so that it is possible to have only one fundamental magnitude. To the c.g.s. or m.k.s. units the fundamental dimensions are mass [ $M$ ], length [ $L$ ], and time [ $T$ ]. In the natural units system the new fundamental dimensions are mass [ $M$ ], action [ $S$ ], and velocity [ $V$ ]. In terms of natural units the lenghts and time are

$$[L] = \frac{[S]}{[M][V]}, \quad [T] = \frac{[S]}{[M][V]^2} \quad (44)$$

and being  $p, q, r$  real numbers the general dimensional relation

$$[M]^p[L]^q[T]^r = [M]^{p-q-r}[S]^{q+r}[V]^{-q-2r} \quad (45)$$

in the c.g.s. or m.k.s. units has the natural units mass dimension  $[M]^n$  with  $n = p - q - r$ . Thereby, for action and velocity we have  $p = 1, q = 2, r = -1$ , and  $p = 0, q = 1, r = -1$  respectively, with  $n = 0$  for both of such magnitudes. The mass or energy,  $E = Mc^2$ , length, and time have the  $p = 1, q = r = 0; q = 1, p = r = 0$ ; and  $r = 1, p = q = 0$  attributions with the respective natural units dimensions  $n = +1, -1, -1$ . In natural units, the energy, mass, and linear momentum have the same dimension, say measured in eV energy unit meanwhile time and distances have the inverted dimension ( $eV^{-1}$ ). What is intended is to know how the world is at distances even shorter or at even higher energies [48]. This makes the present particle accelerators become the microscope solving today lenghts of  $1.9733 \times 10^{-19}$  meters, exactly equivalent to the energy scale of  $1 \text{ TeV} = 10^3 \text{ GeV} = 10^{12} \text{ eV}$ . The *detector physics*, by its turn, as the *neutrino physics* [49] and the *physics of cosmic rays* [50,51] are revealing physics from below the eV scale up to energies even very higher than the TeV scale.

The Planck energy  $E_P$  is associated to the universal gravitation constant  $G$  according to

$$\frac{1}{\sqrt{G}} = E_P \approx 10^{19} \text{ GeV} \quad (46)$$

and numerically,

$$G \approx 5.9 \times 10^{-39} M_{p+} \quad (47)$$

where  $M_{p+}$  is the proton mass. Then, we have the following bound on the cosmological constant,

$$\Lambda \lesssim \frac{10^{-122}}{G} \approx 10^{-84} M_{p+}^2 \simeq 10^{-84} \text{ GeV}^2 \quad (48)$$

but the same order of magnitude is obtained by using the Planck scale of energy

$$\Lambda \lesssim 10^{-122} E_P^2 \simeq 10^{-84} \text{ GeV}^2. \quad (49)$$

The limits realized in the process of desacceleration of the Universe imply

$$|\Lambda| \lesssim 4 \times 10^{-84} \text{ GeV}^2 \simeq 10^{-52} \text{ m}^{-2} \quad (50)$$

on the value of the cosmological constant which in fact coincides with the adimensional quantity  $G\Lambda \lesssim 10^{-122}$ .

Let us now concentrate in the determination of the numerical value [52] of the cosmological constant  $\Lambda_{\text{SM}}$  induced in the spontaneous gauge symmetry breaking  $SU(2) \times U(1)' \rightarrow U(1)$  of the electroweak standard model. Recovering the Zel'dovich relation given in Eq. (41) in which according to Eq. (37) the potential acquires the value given by  $-m^4/(4\lambda)$  we obtain

$$\Lambda_{\text{SM}} = -\frac{2\pi G}{c^4} m^2 \sigma_{\pm}^2 \quad (51)$$

and by using the relation

$$\frac{G_F}{\sqrt{2}} = \frac{1}{2\sigma^2} \quad (52)$$

where  $G_F \simeq 1.02 \times 10^{-5} M_{\text{p}^+}^{-2} \simeq 1.17 \times 10^{-5} \text{ GeV}^{-2}$  is the Fermi universal constant of weak interaction, it results that the cosmological constant is

$$\Lambda_{\text{SM}} = -\frac{\pi G}{2\sqrt{2}c^4G_F} M_H^2 \approx -10^{-33} M_H^2 \quad (53)$$

being proportional to the square of the mass  $M_H^2$  of the Higgs scalar boson. Hence, taking into account the experimental inferior limit  $M_H > 100 \text{ GeV}$  it results

$$\Lambda_{\text{SM}} \sim 10^{54} \Lambda_{\text{obs}}, \quad (54a)$$

with

$$\Lambda_{\text{obs}} = 10^{-122}/G \simeq 10^{-84} \text{ GeV}^2 \quad (54b)$$

comprehending a numerical factor representing the major discordance between theory and experiment of the whole physics. The standard model containing the description of the non-gravitational interactions quantum dynamics and is extremely successful in its concordance with the experiments presents this problem. Every time that the Higgs mechanism [19,20] operates in order to generate masses in a gauge theory it is generated besides a cosmological constant. This is a problem not only belonging to the standard model, but also to the grand unification theories (GUTs) and supersymmetric (SUSY) schemes.

Connected to all these theories, it always appears a hierarchy problem: the energy of the vacuum state of the Universe is zero with a precision of  $10^{-122}$ . In the dynamics of vacuum, where a cosmological phase transition is induced by a scalar field, when we consider the field as a temperature-dependent function, there is a reciprocal mechanism giving a vanishing cosmological constant. The scalar sector of the theory has a vanishing vacuum energy that is induced by itself. This could have occurred in the occasion of a cosmological phase transition in which the Higgs potential is corrected accordingly [53]

$$\mathcal{V}(\phi) = -\mu^2(T) + \lambda\phi^4 \quad (55)$$

being the radiative corrections until first order  $\mathcal{O}(\lambda)$  in the perturbative serie. The mass parameter in terms of temperature is

$$\mu^2(T) = \mu_0^2 - \kappa T^2 \quad (56)$$

where the coefficient  $\kappa$  depends either on  $\phi$  be representing a gauge singlet or any multiplet in the fundamental or adjoint gauge group representations. In general,  $\kappa \sim \lambda$ . The phase transition occurs when the temperature  $T$  is lower than the critical temperature,

$$T_C = (\mu_0^2/\kappa)^{\frac{1}{2}}, \quad T < T_C, \quad (57)$$

when a new state of minimum is created in  $\phi = \sigma$ , which is the vacuum state. In the new phase, the cosmological constant is determined by

$$\mathcal{V}(\phi) = \frac{\mu^4}{4\lambda} = \frac{(\mu_0^2 - \kappa T^2)^2}{4\lambda} \quad (58)$$

which naturally depends on temperature,  $T$ . The idea that a cosmological constant depends on the temperature is not a new one [54]. Nonetheless, the possibility of a vacuum state decaying with temperature throughout the whole history of the Universe [55] is severally suppressed not to interfere with results of the phase of nucleosynthesis [56]. It is possible to build up finite temperature field theories [57] when the dependence of the cosmological constant with temperature is limited only to phase transitions.

## V. THE BAUM–HAWKING–COLEMAN SOLUTION

An attempt to solve the vacuum problem was proposed by E. Baum [59] in 1983, S. W. Hawking [60] in 1984 and S. Coleman [62] in 1988. The BHC solution has two crucial ingredients:

(i) The observable value of the cosmological constant  $\Lambda$  is not absolutely a c-number fundamental parameter. On the contrary, it is necessary to consider Universes with different values of  $\Lambda$  distributed according to a certain distribution,  $\mathcal{P}(\Lambda)$ , in which  $\Lambda$  is promoted to quantum dynamical variable, a q-number.

This presumption was successful by studies of topological solutions such as *baby universes* and *wormholes* [61] which finally led to the fact that all coupling constants, including  $\Lambda$  are dynamical variables.

(ii) It is supposed that the probability distribution  $\mathcal{P}(\Lambda)$  is possible to be determined. Baum [59] and Hawking [60] have proposed that

$$\mathcal{P}(\Lambda) \propto \exp\{-S(\Lambda)\} \quad (59)$$

where the action is

$$S(\Lambda) = -3\pi \frac{M_P}{\Lambda} \quad (60)$$

and thus  $\mathcal{P}(\Lambda)$  is strongly pronounced in  $\Lambda = 0$ ,

$$\lim_{\Lambda \rightarrow 0} \mathcal{P}(\Lambda) = \infty. \quad (61)$$

Coleman [62] proposed

$$\mathcal{P}(\Lambda) \propto \exp\{\exp\{-S(\Lambda)\}\} \quad (62)$$

$$= \exp\{\exp\{3\pi M_P/\Lambda\}\} \quad (63)$$

which has a peak exceptionally more accentuated when  $\Lambda = 0$ .

However, it is not clear until which point the proposition of solution by Baum–Hawking–Coleman [59,60,62] for the cosmological constant is efficient [63], considering that, for instance, the results of  $\mathcal{P}(\Lambda)$  can not be entirely based on the Euclidean functional integration of the quantum gravity. The modified Einstein equations [Eq. (42)] follow from the action

$$S(\Lambda) = (16\pi G)^{-1} \int dx \sqrt{g} (2\Lambda R) \quad (64)$$

which is the most general form of a local action consistent with the invariance principles of general relativity. The BHC solution appeals to the controversial ways of quantum gravity Euclideanization [64] as well as the action  $S(\Lambda)$  is related to the possible non-localities as a result of non-perturbative topological effects as the *wormholes* able to connect two distinct plane Universes or two regions of the same Universe.

## VI. BARYON ASYMMETRY OF THE UNIVERSE

One of the fundamental questions of quantum particle cosmology concerns to the mechanisms which could have generated the baryon asymmetry of the Universe, that is the observed disequilibrium between matter and antimatter. The ratio between the densities of the number of baryons,  $n_b$ , and of photons,  $n_\gamma$ , is given by the estimatives of the parameter [65]

$$\eta \equiv \frac{n_b}{n_\gamma} \simeq 10^{-10 \pm 1} \quad (65)$$

where

$$n_b \simeq 10^{-5} \text{ cm}^{-3} \quad (66a)$$

and

$$n_\gamma \simeq 400 \left( \frac{T}{T_0} \right)^3 \text{ cm}^{-3}. \quad (66b)$$

The data of the COBE-FIRAS satellite [66] indicate a temperature  $T_0 = (2.726 \pm 0.001)$  K of the cosmic background radiation. The cosmic background radiation spectrum adjusts to a thermal bath spectrum with incredible precision. For  $T = T_0$ , the density of the number of photons is  $400 \text{ cm}^{-3}$ .

The asymmetry between baryons and antibaryons can be justified in the following manner: the Universe started with a complete symmetry between matter and antimatter in the standard description of the Big Bang and in the subsequent evolution it was generated a baryon number. In principle, it is possible provided that three conditions be observed:

1. There is a kind of interaction which violates the conservation of the baryon charge at fundamental level;
2. There are interactions which violates the symmetry of charge conjugation  $C$  and the  $CP$  combined symmetry of charge conjugation and parity so that they induce asymmetry among processes involving particles and antiparticles;
3. There are shifts in the thermal equilibrium state of the scalar and vector particles which mediate interactions that violate the conservation of baryon number.

Strictly in the realm of the standard model of the non-gravitational interactions it is not possible to generate the violation of the conservation of the baryon number. Nevertheless, there are instanton [67] kind solutions of the corresponding equations of motion. It is associated to the instantons a quantum number, the topological charge, and induces interactions between quarks and leptons as

$$(u + u + d) + (c + c + s) + (t + t + b) \rightarrow e^+ + \mu^+ + \tau^+ \quad (67)$$

in which there are three quarks comprehended between parentheses with different  $SU(3)_c$  color charges but that are elements of the same doublet of weak isospin. However, such transitions are suppressed by a factor  $\exp\{-4\pi/\alpha_W\}$  where  $\alpha_W = \alpha/\sin^2\theta_W$  and

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137.035\,989\,5(61)} \quad (68)$$

is the fine structure constant and  $e = 1.602\,177\,33(49) \times 10^{-19}$  C is the unit of electric charge. Therefore,

$$\exp\{-4\pi/\alpha_W\} \simeq \exp\{-398\} \simeq 10^{-172} \quad (69)$$

which is 50 orders of magnitude even smaller than the bound on the cosmological constant,  $G\Lambda \lesssim 10^{-122}$  (!).

## VII. CASIMIR EFFECT

There are phenomena of quantum nature [68] related to the presence of fluctuations in the vacuum state of the quantized fields. One of the recognizable predictions of the quantum electrodynamics (QED) was announced in 1948 by Hendrik B. Casimir [69] who proposed the effect where a variation of the vacuum energy density produce an attractive force between two plane and electric conductor plates. For two metallic parallel plates, with area  $A$  and separated by a distance  $d$ , the attractive Casimir force,  $F(d)$ , by unit of area,  $A$ , is [32]

$$\frac{F(d)}{A} = \frac{\pi^2}{240} \frac{\hbar c}{d^4} \propto \frac{1}{d^4}. \quad (70)$$

This attraction was experimentally confirmed, for the first time, after ten years [70]. For plates of  $1\text{ cm}^2$  of area and being  $d = 0.5\text{ }\mu\text{m}$ , the measured value of the force was  $\sim 2 \times 10^{-6}\text{ N}$  that, in fact, agrees with Eq. (70). Recently, the effect of the Casimir force

was shown in the scale between 0.6 and 6  $\mu\text{m}$ . The pressure of the vacuum state between two very near conductor surfaces is considered conclusively demonstrated [71] due to the fluctuations of the fundamental quantum state.

This kind of force is part of a group of effects due to variations of the energy density of the vacuum state now collectively called Casimir forces [72]. The Lamb [73] shift verified in 1947 is a result of the vacuum energy shift of the hydrogen atom. Hence, the quantum vacuum state is much more than empty space.

### VIII. ANTHROPOIC PRINCIPLE

The anthropic principle [74] states that the Universe is the way it is because otherwise there would not be anyone to ask why it is this way. Many of the aspects of the Universe would be determined by the condition that there is intelligent life in it. One of these aspects [75] is referent to the dimensions of the Universe since it was smaller, with a larger matter density, it would already had suffered a gravitational collapse before intelligent life had evolved. Another aspect refers to the lifetime of the proton whose experimental bound is larger than  $10^{16}$  years because on the contrary living beings would not survive to the effects of the ionizing particles produced by the proton decay in their organisms. There is almost one mol of stars in the Universe. How many of them there had all the conditions for life to become intelligent? A lot of them? May be. Besides, it is possible that ‘there is something unique relatively to the Man and to the planet in which he lives’ according to the geneticist T.G. Dobzhansky (1900–1975).

The anthropic principle [74] in a version a lot stronger states that the natural laws are complete only in the case that there is intelligent life once quantum mechanics would not have sense without the observer. Weinberg [47] however in relation to this strong version states that ‘despite the fact that science is clearly impossible without scientists it is not clear that the Universe is impossible without science.’

Finally, the weak anthropic principle searchs for an explanation of what are the possible eras and parts of the Universe where we could be calculating which eras and parts of the Universe we can live and illustrates the first use of anthropic arguments in modern physics with the Dicke [75] solution to the problem proposed by Dirac. Still in 1937 Dirac [76] realized the combination with dimension of time of physical universal constants providing the age of the Universe,

$$t_U = \frac{\hbar}{G c} \frac{\alpha}{m_{e^-}^2 m_{p^+}} \simeq 4.5 \times 10^{10} \text{ years.} \quad (71)$$

Let us observe that the time scale goes from the Planck scale

$$t_P = (\hbar G / c^5)^{\frac{1}{2}} \simeq 10^{-44} \text{ sec} \quad (72)$$

up to the scale determined by the Hubble [77] constant,  $H_0$ , the fundamental constant of cosmology, which establishes the age of the Universe,

$$t_U = H_0^{-1} \simeq 10^{10} \text{ years} \simeq 10^{17} \text{ sec} \quad (73)$$

so that the time scale has the bounds

$$t_P \simeq 10^{-44} \text{ s} \leq t \leq t_U \simeq 10^{17} \text{ s} \quad (74)$$

determined by the Planck and Hubble scales, respectively. According to the modified Einstein equations, Eq. (42), involving the  $\Lambda g_{\mu\nu}$  term of the cosmological constant, the universal expansion law of the scale factor  $R(t)$  of the Universe is given by

$$H^2 \equiv \left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G}{3} \rho_M + \frac{\Lambda}{3} - \frac{k}{R^2}, \quad (75)$$

where  $\dot{R} = dR(t)/dt$ ,  $\rho_M$  is the matter density of the Universe;  $k = -1, 0, +1$  to Universes that are curved open, flat, and curved closed and, finally,  $H$  is the Hubble constant whose present value is

$$H_0 = 100 h_0 \text{ km s}^{-1} \text{ Mpc}^{-1} \quad (76)$$

with  $1 \text{ Mpc} (\text{megaparsec}) \simeq 3.1 \times 10^{19} \text{ km} \simeq 3 \text{ light-year}$ . In a natural unit,

$$H_0 = 2.13 h_0 \times 10^{-42} \text{ GeV} \quad (77)$$

with an adimensional ignorance parameter in the interval  $0.5 < h_0 < 0.87$ . The Supernova Cosmology Project data [78] gives the value

$$H_0 = (63.1 \pm 3.4 \pm 2.9) \text{ km s}^{-1} \text{ Mpc}^{-1} \quad (78)$$

and a theoretical deduction [79] gives

$$H_0 = 61.4 \text{ km s}^{-1} \text{ Mpc}^{-1}. \quad (79)$$

The Hubble law above contains three terms determining the expansion of the Universe. The first one is the common term of matter, the second one is that of the cosmological constant and, finally, the last one is the space-time curvature term.

The Dicke solution to the Dirac problem indicates that the question of the age of the Universe can appear only when the conditions for the existence of life are appropriate and correct. The Universe should have enough age to some stars having completed their permanence in the principal sequence and produced heavy chemical elements necessary to the existence and maintenance of life. Otherwise, other stars should be young enough so that they are still producing energy, i.e., light and heat for life, by means of thermonuclear reactions.

Dirac argued that if the connection established in Eq. (71) were anyhow fundamental as the age of the Universe increases linearly with the time  $t$ , at least one constant on the Eq. (71) should vary with the time and conjectured that the universal constant  $G$  of gravitation varies according to  $1/t$ . In 1985 Zee [80] applied the same procedure to the cosmological constant based in the condition  $G\Lambda \lesssim 10^{-122}$  so that, if  $G = G(t) \propto 1/t$ , then  $\Lambda(t) \propto t$ . According to Weinberg, it is perfectly reasonable to apply anthropic considerations in order to know in which era or part of the Universe we could exist and, therefore, what values of the cosmological constant we could observe.

## IX. CONCLUSIONS

Although Einstein called his attempt of introducing a cosmological constant in their equations of general relativity to keep the Universe static the worst mistake of his whole life [81], he did not suspect that a so large cosmological constant would be induced in any theory with spontaneous gauge symmetry breaking. The cosmological constant remains an essential quantity, either of cosmology or of high energy physics [82]. In the case of cosmology due to the strong constraint  $G\Lambda \lesssim 10^{-122}$  and for the high energy physics, in the context of quantum field theories with local gauge symmetries as the standard model of non-gravitational interactions, to a cosmological constant corresponding to the energy density associated to the vacuum state in the process of spontaneous gauge symmetry breaking which is not canceled. The vacuum is a busy place in any quantum theory and should gravitate [83–85].

The seriousness of the vacuum state energy problem [86] conduced a lot of physicists to believe that it should have exist some mechanism of cancellation or that quantum considerations of cosmology would justify a vanishing value. The problem is that it is not possible to find any symmetry that warrant an identically zero value [87] and arguments of quantum cosmology are still based in the quantum Euclidean gravity. Even the Peccei–Quinn [88] symmetry results in an imperfect cancellation. A cancellation mechanism is viable [89] in the leptoquark-bilepton flavor dynamics [90]. There is also the possibility of a probability distribution in which the cosmological constant takes different values in cosmological theories with a large number of sub-Universes with different terms in the wave function of the Universe [91].

Finally, it is also considered the intriguing possibility of the energy content of the vacuum state is zero but, in the moment, we would be in a phase transition in which the Universe is in the state of false vacuum [92]. The energy scale of this transition corresponds to  $(10^{-46} \text{ GeV}^4)^{\frac{1}{4}} \simeq 3 \times 10^{-3} \text{ eV}$  which can be near to the value of lightest neutrino mass, which would also solve the solar neutrino problem [93].

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